4.3 Shear Force and Bending Moment Diagrams:

4.3.1 Procedure for Analysis:

The following step-by-step procedure can be used for determining the member end forces as well as the shears, bending moments, and axial forces in members of plane statically determinate frames:

1. Check for static determinacy. Using the procedure described in the preceding section, determine whether or not the given frame is statically determinate. If the frame is found to be statically determinate and stable, proceed to step 2. Otherwise, end the analysis at this stage.
2. Determine the support reactions. Draw a free-body diagram of the entire frame, and determine reactions by applying the equations of equilibrium and any equations of condition that can be written in terms of external reactions only (without involving any internal member forces). For some internally unstable frames, it may not be possible to express all the necessary equations of condition exclusively in terms of external reactions; therefore, it may not be possible to determine all the reactions. However, some of the reactions for such structures can usually be calculated from the available equations.
3. Determine member end forces. It is usually convenient to specify the directions of the unknown forces at the ends of the members of the frame by using a common structural (or global) XY coordinate system, with the X and Y axes oriented in the horizontal (positive to the right) and vertical (positive upward) directions, respectively. Draw free-body diagrams of all the members and joints of the structure. These free-body diagrams must show, in addition to any external loads and support reactions, all the internal forces being exerted upon the member or the joint. Remember that a rigid joint is capable of transmitting two force components and a couple, a hinged joint can transmit two force components, and a roller joint can transmit only one force component. If there is a hinge at an end of a member, the internal moment at that end should be set equal to zero. Any load acting at a joint should be shown on the free-body diagrams of the joint, not at the ends of the members connected to the joint. The senses of the member end forces are not known and can be arbitrarily assumed. However, it is usually convenient to assume the senses of the unknown forces at member ends in the positive X and Y directions and of the unknown couples as counterclockwise. The senses of the internal forces and couples on the free-body diagrams of joints must be in directions opposite to those assumed on the member ends in accordance with Newton's third law. Compute the member end forces as follows:
    a. Select a member or a joint with three or fewer unknowns.

b. Determine the unknown forces and moments by applying the three equations of equilibrium to the free body of the member or joint selected in step 3(a).
c. If all the unknown forces, moments, and reactions have been determined, then proceed to step 3(d). Otherwise, return to step 3(a).
d. Since the support reactions were calculated in step 2 by using the equations of equilibrium and condition of the entire structure, there should be some equations remaining that have not been utilized so far. The number of leftover equations should be equal to the number of reactions computed in step 2. Use these remaining equations to check the calculations. If the analysis has been carried out correctly, then the remaining equations must be satisfied.

For some types of frames, a member or a joint that has a number of unknowns less than or equal to the number of equilibrium equations may not be found to start or continue the analysis. In such a case, it may be necessary to write equilibrium equations in terms of unknowns for two or more free bodies and solve the equations simultaneously to determine the unknown forces and moments.

4. For each member of the frame, construct the shear, bending moment, and axial force diagrams as follows:
   a. Select a member (local) *xy* coordinate system with origin at either end of the member and x axis directed along the centroidal axis of the member. The positive direction of the y axis is chosen so that the coordinate system is right-handed, with the *z* axis pointing out of the plane of the paper.
   b. Resolve all the external loads, reactions, and end forces acting on the member into components in the x and y directions (i.e., in the directions parallel and perpendicular to the centroidal axis of the member). Determine the total (resultant) axial force and shear at each end of the member by algebraically adding the *x* components and y components, respectively, of the forces acting at each end of the member.
   c. Construct the shear and bending moment diagrams for the member by using the procedure described before. The procedure can be applied to non-horizontal members by considering the member end at which the origin of the *xy* coordinate system is located as the left end of the member (with x axis pointing toward the right) and the positive y direction as the upward direction.
   d. Construct the axial force diagram showing the variation of axial force along the length of the member. Such a diagram can be constructed by using the method of sections. Proceeding in the positive x direction from the member end at which the origin of the *xy* coordinate system is located, sections are passed after each successive change in loading along the length of the member to determine the equations for the axial force in terms of x. According to the

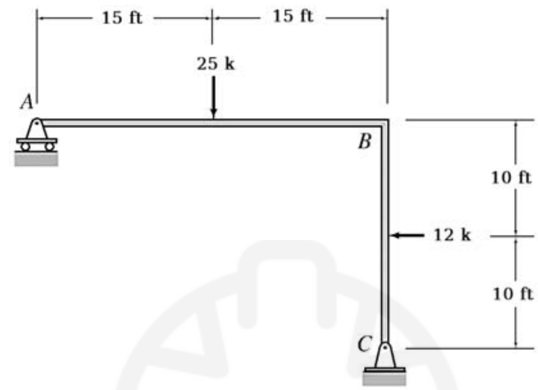
sign convention adopted before, the external forces acting in the negative x direction (causing tension at the section) are considered to be positive. The values of axial forces determined from these equations are plotted as ordinates against x to obtain the axial force diagram.

5. Draw a qualitative deflected shape of the frame. Using the bending moment diagrams constructed in step 4, draw a qualitative deflected shape for each member of the frame. The deflected shape of the entire frame is then obtained by connecting the deflected shapes of the individual members at joints so that the original angles between the members at the rigid joints are maintained and the support conditions are satisfied. The axial and shear deformations, which are usually negligibly small as compared to the bending deformations, are neglected in sketching the deflected shapes of frames.

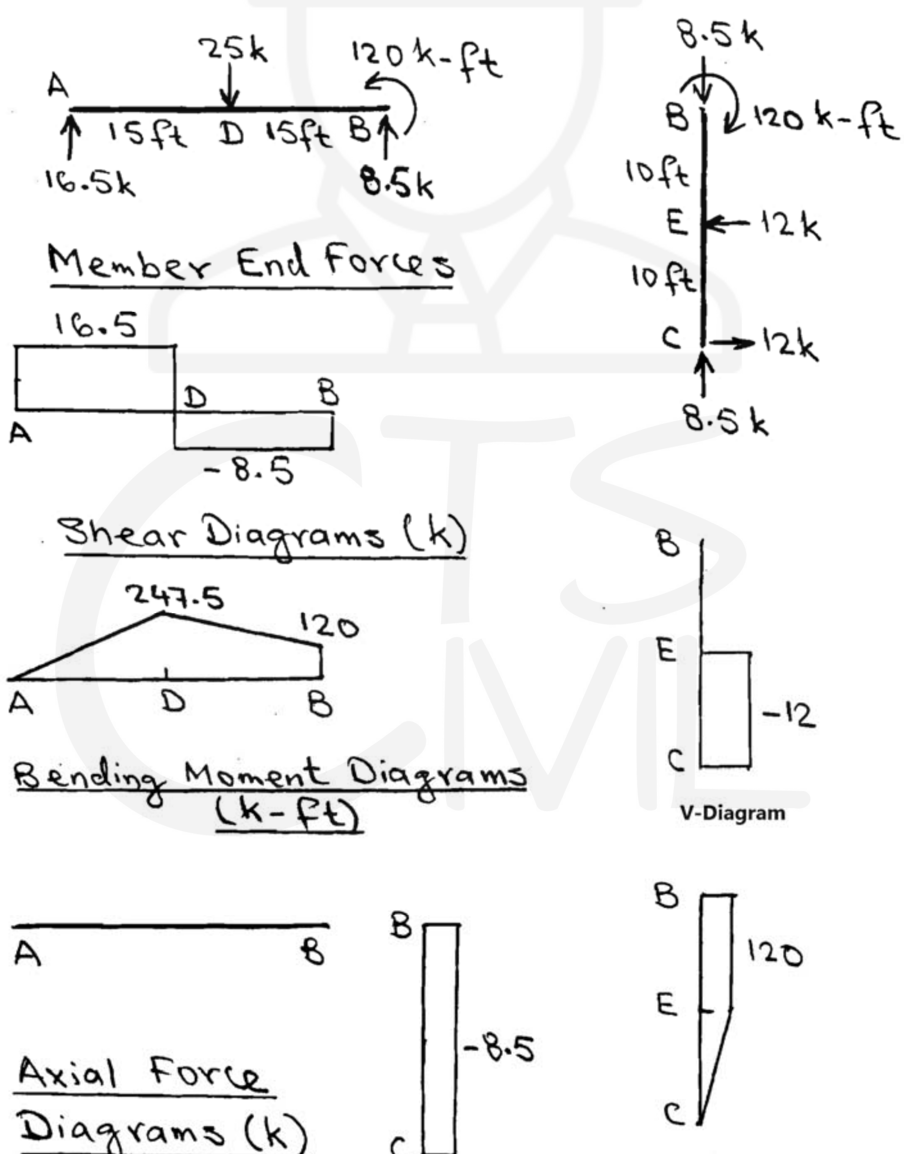
It should be noted that the bending moment diagrams constructed by using the procedure described in step 4(c) will always show moments on the compression sides of the members. For example, at a point along a vertical member, if the left side of the member is in compression, then the value of the moment at that point will appear on the left side. Since the side of the member on which a moment appears indicates the direction of the moment, it is not necessary to use plus and minus signs on the moment diagrams. When designing reinforced concrete frames, the moment diagrams are sometimes drawn on the tension sides of the members to facilitate the placement of steel bars used to reinforce concrete that is weak in tension. A tension-side moment diagram can be obtained by simply inverting (i.e., rotating through 180 degrees about the member's axis) the corresponding compression-side moment diagram. Only compression-side moment diagrams are considered in this text.

4.3.2 Examples:

Example (1):



Solution:



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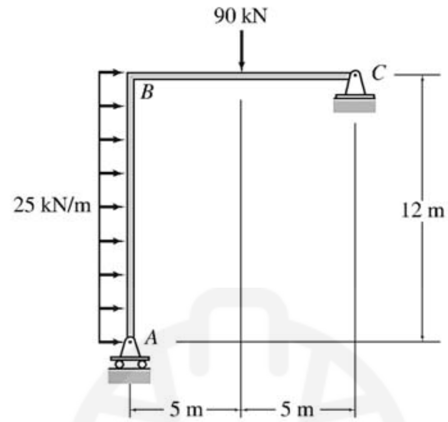
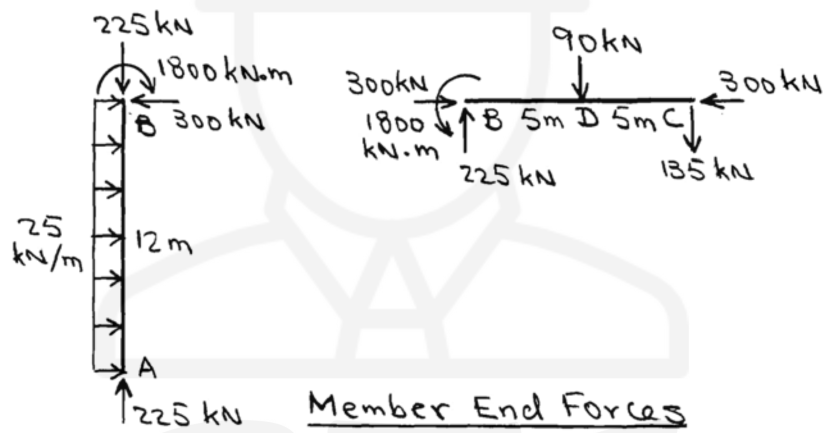
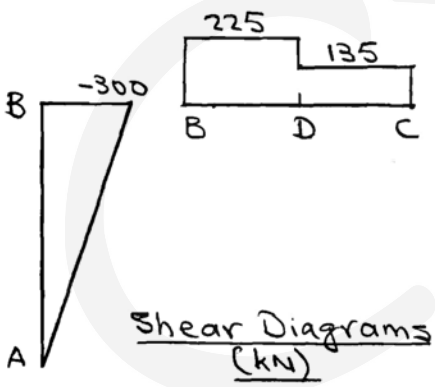
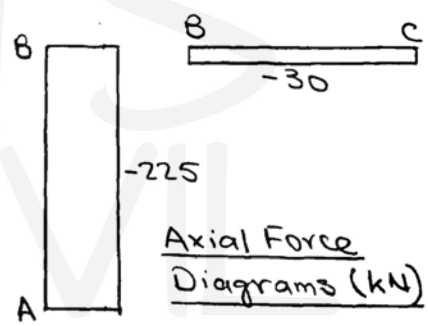
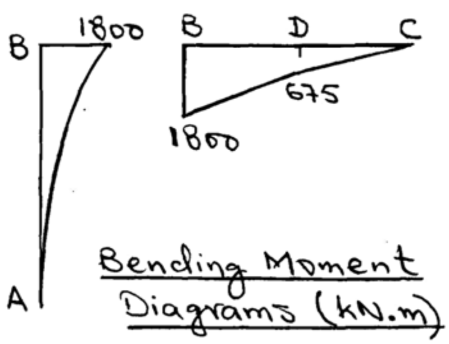


Solution:









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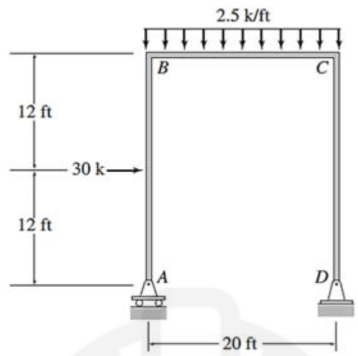

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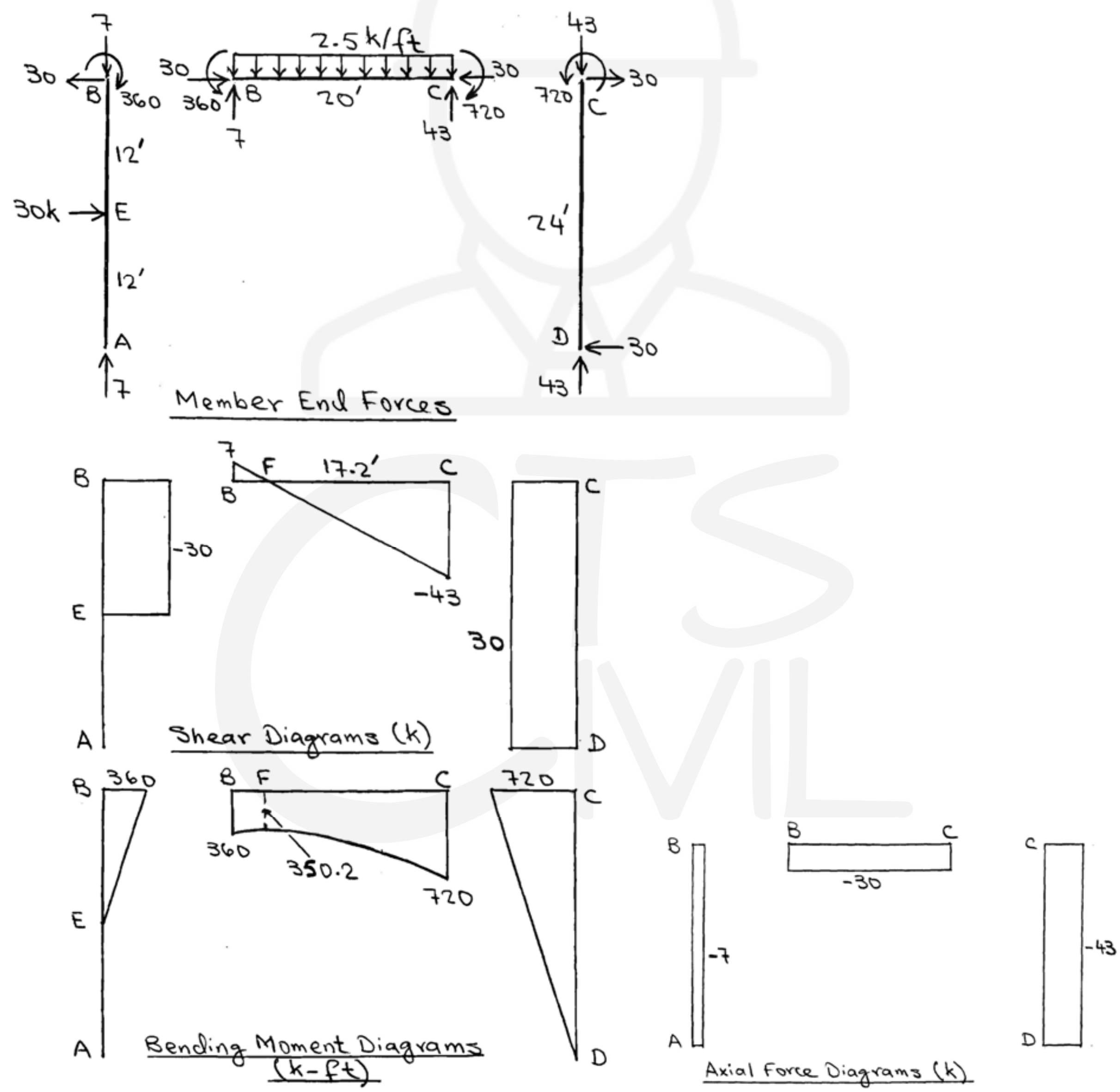

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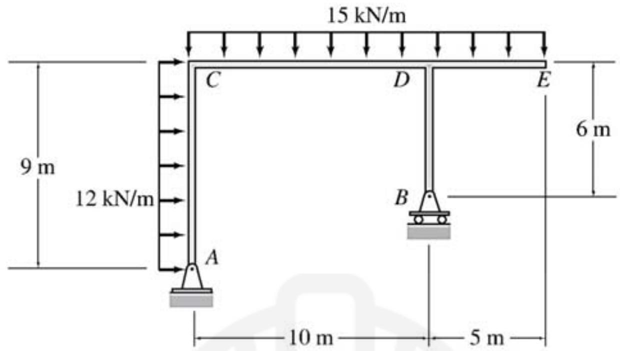
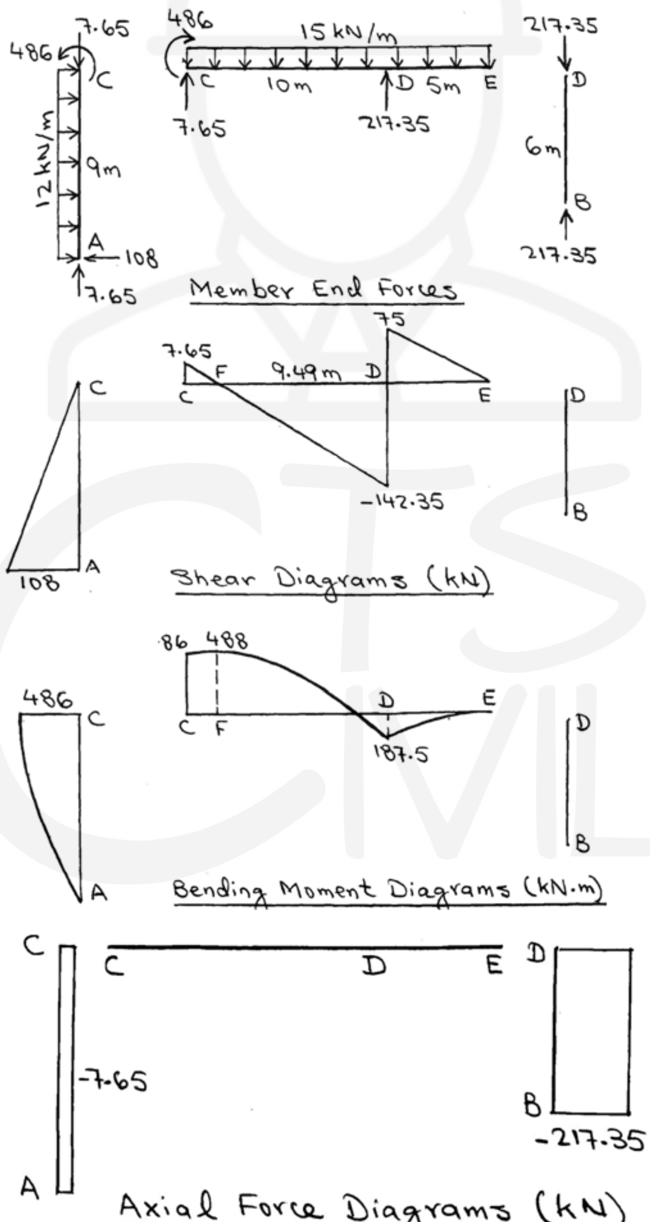


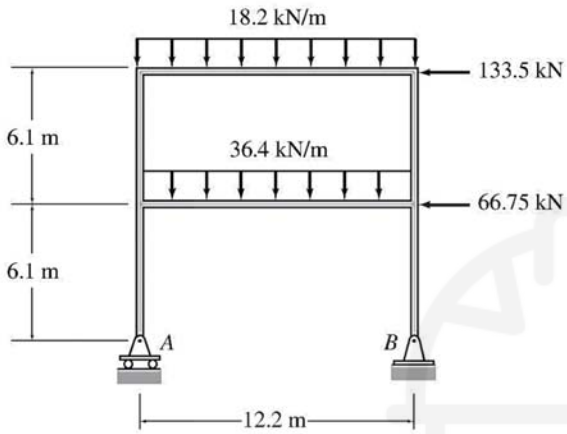
Solution:



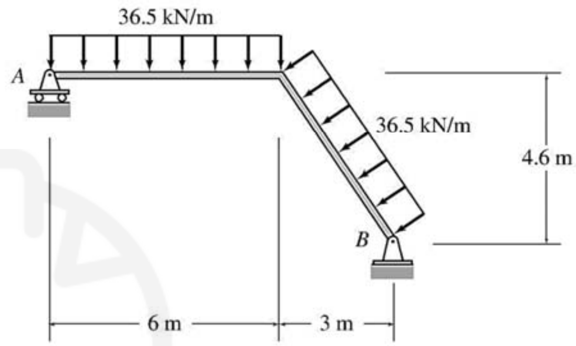
4.4 Problems:

Calculate the supports reactions for the following frames:

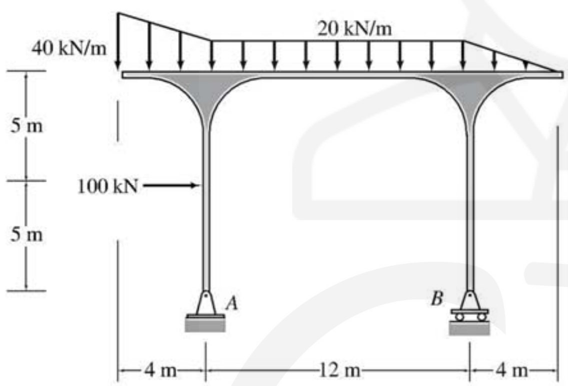
Frame 1:



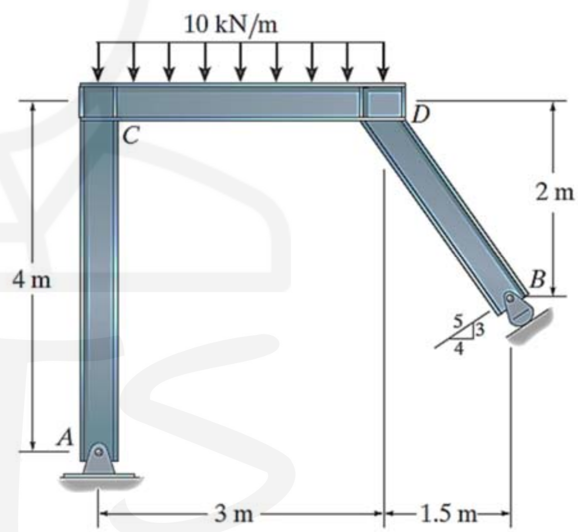
Frame 2:



Frame 3:

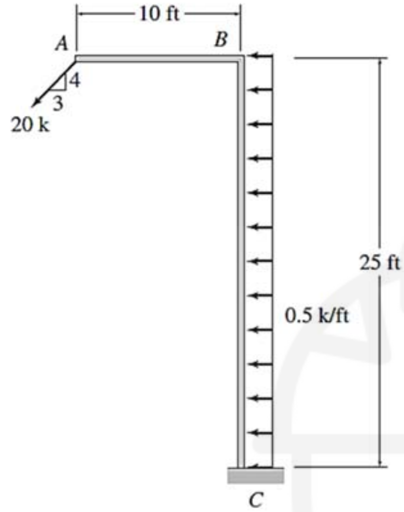


Frame 4:



Draw the shear force and bending moment diagrams for the following frames:

Frame 5:



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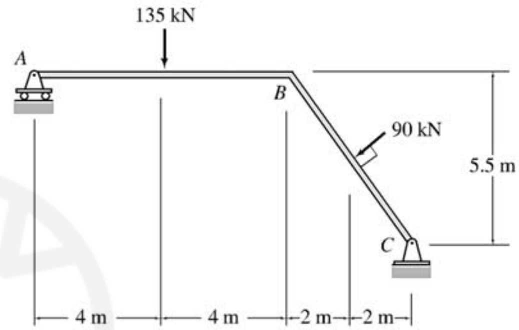

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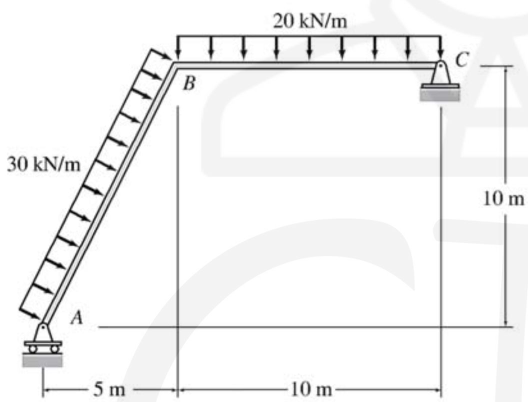

Frame 8: